

## Major areas tested on 6<sup>th</sup> grade MEAP (from the 5<sup>th</sup> grade GLCEs)

- 1) multi-digit multiplication and division involving remainders (green)
- 2) add and subtract fractions (blue);
- 3) problem solving with decimal numbers (orange)
- 4) convert measurements within a given system (purple) – concepts and multiplication practice
- 5) measure angles and solve angle problems (pink)

GLCEs in grey will not be tested on the MEAP because they moved to a higher grade in the CCSS.

GLCE	GLCE Grade	CALC	GLCE Descriptor	Match to CCSS	
N.MR.05.02	5th	N	Know division of whole numbers in form $a = bq + r$	5th	
N.MR.05.01	5th	N	Understand the meaning of division with & w/o remainders	5th	core
N.MR.05.03	5th	N	Write mathematical statements involving division	5th	core
N.FL.05.04	5th	N	Multiply a multi-digit number by a two-digit number	5th	core
N.FL.05.05	5th	Y	Solve problems involving $\times$ and $\div$ of whole numbers	5th	core
N.FL.05.06	5th	N	Divide up to a 4-digit number by a two-digit number	5th	
N.MR.05.07	5th	Y	Find prime factorization of #s, show exponentially	6th	core
N.ME.05.08	5th	N	Understand the relative magnitude base-10 system	5th	
N.ME.05.09	5th	Y	Understand percentages as parts out of 100	6th	
N.ME.05.10	5th	N	Understand & show fractions as a statement of $\div$	5th	
N.ME.05.11	5th	N	Compare two fractions using common denominators	5th	
N.ME.05.12	5th	N	Multiply two unit fractions using area model	5th	
N.MR.05.13	5th	N	Divide using fractions and whole numbers	5th	
N.FL.05.14	5th	N	Add and subtract fractions with unlike denominators	5th	core
N.FL.05.18	5th	N	Write statements involving + and - of fractions	5th	core
N.MR.05.19	5th	N	Solve contextual problems involving +/- fractions	5th	core
N.FL.05.20	5th	Y	Solve applied problems using fractions [& decimals]	5th	core
N.MR.05.21	5th	N	Solve for the unknown in equations with fractions	5th	
N.MR.05.15	5th	N	$\times$ a whole number by powers of 10, identify patterns	5th	
N.FL.05.20	5th	Y	Solve applied problems using [fractions &] decimals	5th	core
N.FL.05.16	5th	Y	Divide numbers by powers of 10	5th	core
N.MR.05.17	5th	N	Multiply decimals to 100ths by whole numbers	5th	
N.MR.05.22	5th	Y	Express fractions and decimals as percentages	6th	
N.ME.05.23	5th	Y	Express ratios in the forms a to b, a:b, a/b	6th	
M.UN.05.01	5th	Y	Know equivalence of 1 liter, 1000 ml and 1000 cc	5th	
M.UN.05.02	5th	N	Know the units of measure of volume	5th	
M.UN.05.03	5th	Y	Compare relative sizes of cubic measures	5th	
M.UN.05.04	5th	N	Convert measurements within a given system	5th	core
M.PS.05.05	5th	N	Show relationships between areas of polygons	7th	core
M.TE.05.06	5th	Y	Know how to use the area formula of a triangle	7th	core
M.TE.05.07	5th	Y	Know how to use area formula for a parallelogram	7th	
M.PS.05.10	5th	Y	Solve volume problems of rectangular prisms	5th	
G.TR.05.01	5th	N	Associate an angle with a certain amount of turning	5th	
G.GS.05.03	5th	N	Identify angles on a straight line & vertical angles	5th	

G.GS.05.02	5th	N	Measure angles with a protractor and classify	5th	core
G.GS.05.04	5th	N	Find unknown angles in problems	5th	core
G.GS.05.05	5th	N	Know straight angle and angles surrounding a point	5th	core
G.GS.05.06	5th	Y	Know interior angles of a triangle & quadrilateral	7th	core
G.GS.05.07	5th	Y	Find unknowns using properties of triangles, quads.	7th	core
D.RE.05.01	5th	Y	Read and interpret line graphs, and solve problems	5th	
D.RE.05.02	5th	Y	Construct line graphs from tables of data	5th	
D.AN.05.03	5th	Y	Given set of data, find & interpret mean, mode	6th	core
D.AN.05.04	5th	Y	Solve multi-step problems involving means	6th	

## Multi-digit multiplication

Students learn this in 4<sup>th</sup> grade (3 digits by 2 digits) first by using an area model to help understand the distributive property, then by using an algorithm. In 5<sup>th</sup> grade they extend what they learned to 4 or 5 digits by 2 digits. There are several alternative algorithms that are simpler for many students, including the partial product method and the lattice method. Either should be taught to students who are error-prone when using the traditional algorithm. See the 5<sup>th</sup> grade MEAP review packet for information on these alternative algorithms.

## Multi-digit division

Once students understand the concepts of division and can perform divisions within 100, they should begin to estimate divisions of larger numbers, starting with ones that are relatively easy to estimate and working towards ones that require more thought. Here's a sample of problems:  $96 \div 6$ ,  $120 \div 6$ ,  $144 \div 6$ ,  $300 \div 6$ ,  $356 \div 6$ ,  $100 \div 15$ ,  $400 \div 15$ .

An alternative algorithm for long division is the repeated subtraction method. As students get more practice with this, they become more proficient by making larger initial estimates. This helps with their ability to estimate the actual answer. Repeated subtraction is more error-free than the traditional algorithm for many students. Examples are shown below.

However, students should not waste time and effort doing too many long divisions without the aid of a calculator. Once they get the idea with two digit divisors, and understand the concept of multi-digit division, they can simply use a calculator, as adults do. They should continue to make estimates of real-world division situations involving larger numbers.

Repeated subtraction examples:

$$\begin{array}{r}
 15 \overline{) 390} \\
 \underline{-150} \quad 10 \\
 240 \\
 \underline{-150} \quad 10 \\
 90 \\
 \underline{-30} \quad 2 \\
 60 \\
 \underline{-30} \quad 2 \\
 30 \\
 \underline{-30} \quad 2 \\
 \hline
 26
 \end{array}$$

Sometimes it helps students to make a table of simple multiples first:

$15 \times 10 = 150$   
 $15 \times 20 = 300$  this is twice  $15 \times 10$   
 $15 \times 5 = 75$  this is half of  $15 \times 10$   
 $15 \times 25 = 375$  this is  $(15 \times 20) + (15 \times 5)$

In the second example, students can take away multiples of 10 if that's easier for them, until they see patterns emerge. One pattern (which a student may mention to other students) is taking half of  $100 \times 46$  (in this case).

The purpose of doing divisions without a calculator is to learn the concepts behind the operation. But once students understand the concepts behind long division, they should be allowed to use calculators for large calculations.

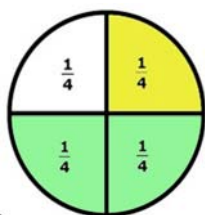
$$\begin{array}{r}
 46 \overline{) 3129} \\
 \underline{-2300} \quad 50 \quad \text{This is half of } 100 \times 46 \\
 829 \\
 \underline{-460} \quad 10 \\
 369 \\
 \underline{-230} \quad 5 \quad \text{This is half of } 10 \times 46 \\
 139 \\
 \underline{-92} \quad 2 \quad \text{Doubling is easy} \\
 47 \\
 \underline{-46} \quad 1 \\
 \hline
 R 1 \quad 68
 \end{array}$$

## Adding and Subtracting Fractions

Adding and subtracting fractions follows a progression:

1. adding and subtracting fractions with like denominators (to learn the basic concept about adding or subtracting a number of same-size pieces of the same whole)
2. adding and subtracting fractions where one denominator is a multiple of the other (because scaling up to find a common denominator is easy)
3. adding and subtracting fractions where the two denominators are not multiples (but use denominators of 12 or less).

The other important progression for learning about fraction operations is the C-R-A progression, concrete to representational to abstract (or objects to pictures to symbols). Start with fraction manipulatives (concrete) like fraction circles to model adding and taking away parts of a whole.



Then move to drawings (which can look just like the manipulatives).



Then write the number sentence:  $\frac{1}{4} + \frac{2}{4} = \frac{3}{4}$ . It is now clear that the procedure is to add the numerators, because they represent how many pieces are being added.

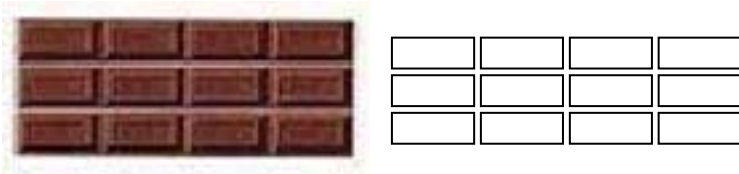
# Adding and Subtracting Fractions

A workbook for students

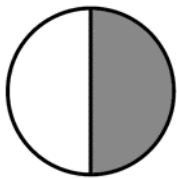
## Part 1

### Equivalent Fractions

Jackie has  $\frac{1}{3}$  of a Hershey bar. Steven has  $\frac{4}{12}$  of a Hershey bar.  
Who has more?



Use fraction circle pieces to figure out how many different ways you can make  $\frac{1}{2}$  out of different pieces.



Write at least three combinations of other fractions that are the same as  $\frac{1}{2}$ :

$$\frac{1}{2} =$$

$$\frac{1}{2} =$$

$$\frac{1}{2} =$$

Two fractions that represent the same amount of the whole are called **equivalent fractions**.

Find two equivalent fractions for each of these. Use fraction circles or drawings if you want

$$\frac{3}{4} =$$

$$\frac{1}{4} =$$

$$\frac{1}{3} =$$

$$\frac{2}{3} =$$

Do you see a mathematical procedure you could use to find equivalent fractions? Explain what the procedure might be.

Mathematically, you can **scale up**  $\frac{1}{2}$  to each of the other fractions by doubling, tripling or quadrupling the numerators and denominators. Figure out another fraction that is equivalent to  $\frac{1}{2}$ .

Find equivalent fractions that have smaller denominators for each of these:

$$\frac{6}{8} =$$

$$\frac{6}{12} =$$

$$\frac{4}{10} =$$

$$\frac{3}{9} =$$

Do you see a mathematical procedure you could use to find equivalent fractions that have smaller denominators? You would **scale down** in this case, because the new fractions use proportionally smaller numbers.

## Part 2

### Adding and subtracting fractions with the same denominator

Think about this: You have  $\frac{2}{6}$  of a pizza. Is this less than  $\frac{1}{2}$  or more than  $\frac{1}{2}$ ? Use fraction circles to help figure this out. Explain your answer.

**1. Your class had a pizza party.  $\frac{3}{8}$  of one pizza was left over, and  $\frac{4}{8}$  of another pizza was left over. You put them both into one box.**

- Do you have more than 1 whole pizza, or less than 1 whole pizza?

Explain your answer. Use fraction circle pieces or drawings to help explain.

- How much pizza do you have altogether?  $\frac{3}{8} + \frac{4}{8} =$

Does this problem make sense if you think of each eighth of the pizza as one slice? Is this how many slices you have altogether? 3 slices + 4 slices = \_\_\_ slices

This does make sense because the denominator of a fraction tells how big each piece is. Each pizza is cut into 8 pieces, so each piece is one eighth of the whole.  $\frac{3}{8}$  of the pizza is the same as 3 slices of the pizza.  $\frac{4}{8}$  is 4 slices. You're just adding slices.

Try this with a slightly different problem:

**2. A cake recipe requires  $\frac{3}{5}$  cup of sugar for the frosting and  $\frac{1}{5}$  cup of sugar for the cake. How much sugar is that altogether?**

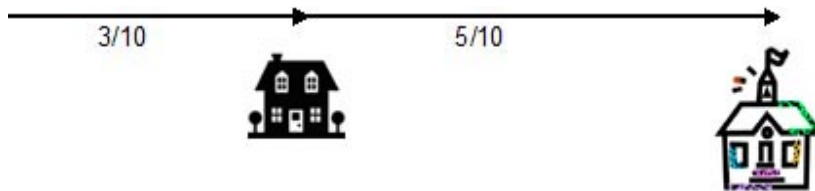
Explain your answer. Use drawings, fraction bars or fraction circles to explain.

Use fraction circles, fraction bars or drawings to find

$$\frac{1}{3} + \frac{4}{3}$$

$$1\frac{2}{3} + \frac{2}{3}$$

3. You walk  $\frac{3}{10}$  of a mile to your friend's house, and then  $\frac{5}{10}$  of a mile to school. How far did you walk altogether?



Describe a mathematical procedure you can use to add fractions with the same denominators.



### Part 3

## Adding and subtracting fractions with different denominators

4. There is  $\frac{3}{8}$  of a pizza in one box and  $\frac{1}{4}$  of a pizza in another box.

➤ If you put the leftover pizza into one box, would you have more than  $\frac{1}{2}$  or less than  $\frac{1}{2}$  of a whole pizza?

Explain your thinking. Use a drawing or fraction circles if you want.

➤ How much do you have altogether?  $\frac{3}{8} + \frac{1}{4} =$

What did you do to find the answer?

Try your procedure again to add or subtract these fractions:

$$\frac{3}{4} + \frac{5}{8}$$

$$\frac{2}{3} - \frac{1}{6}$$

Try these problems:

5.  $\frac{1}{10}$  of the M&M's in a bag are red and  $\frac{1}{5}$  are blue. What fraction of all the M&M's are red and blue? \_\_\_\_\_ What fraction of the M&M's are NOT red or blue?

6. You give  $\frac{1}{3}$  of a pan of brownies to Susan and  $\frac{1}{6}$  of the pan of brownies to Patrick. How much of the pan of brownies did you give away? How much do you have left?

7. You go out for a long walk. You walk  $\frac{3}{4}$  mile and then sit down to take a rest. Then you walk  $\frac{3}{8}$  of a mile. How far did you walk altogether?

8. Pam walks  $\frac{7}{8}$  of a mile to school. Paul walks  $\frac{1}{2}$  of a mile to school. How much farther does Pam walk than Paul?
9. A school wants to make a new playground by cleaning up an abandoned lot that is shaped like a rectangle. They give the job of planning the playground to a group of students. The students decide to use  $\frac{1}{4}$  of the playground for a basketball court and  $\frac{3}{8}$  of the playground for a soccer field. How much is left for the swings and play equipment? Draw a picture to show this.

Explain a procedure you can use to add or subtract fractions that have different denominators.

#### Part 4

### Adding and subtracting with denominators that are not multiples

**10. Marty made two types of cookies. He used  $\frac{2}{3}$  cup of sugar for one recipe and  $\frac{1}{4}$  cup of sugar for the other.**

- Is the total amount of sugar greater than  $\frac{1}{2}$  cup or less than  $\frac{1}{2}$  cup?
- Is the total amount greater than 1 cup or less than 1 cup?

Explain your answers.

Some people would say that  $\frac{2}{3} + \frac{1}{4} = \frac{3}{7}$ .

- a) Why would they say this?
- b) Do you agree?
- c) How would you solve this problem? Work with fraction circles to figure this out. Record your answer below.

$$\frac{2}{3} + \frac{1}{4} =$$

- How is the denominator of the answer related to the two denominators in the problem?

Try these:

$$\frac{1}{3} + \frac{1}{2} =$$

$$\frac{1}{6} + \frac{3}{4} =$$

$$\frac{2}{6} + \frac{1}{5} =$$

**If you can add fractions, you can subtract them also. Try some as a review:**

1. After a party,  $\frac{5}{8}$  of the cake is left over. That night, big brother eats  $\frac{2}{8}$  of the cake. How much is left over after that?
2. You have  $7\frac{5}{8}$  feet of yarn to make a bracelet. You only use  $4\frac{1}{8}$  yards for the bracelet. How much yarn is left over?
3. Susan swims a race in  $29\frac{3}{10}$  seconds. Patty swims the race in  $33\frac{9}{10}$  seconds. How much faster was Susan than Patty?
4. A pitcher contains  $2\frac{3}{4}$  pints of orange juice. After you pour  $\frac{5}{8}$  of a pint into a glass, how much is left in the pitcher?
5.  $\frac{7}{8} - \frac{1}{2} =$
6.  $\frac{5}{6} - \frac{1}{4} =$
7.  $\frac{7}{10} - \frac{3}{4} =$

## Decimal numbers

Students are introduced to the idea of tenths and hundredths as decimal numbers in 4<sup>th</sup> grade, while the 5<sup>th</sup> grade curriculum extends and solidifies that understanding.

The main 5<sup>th</sup> grade core GLCE is:

- Solve applied problems involving [fractions and] decimals; include rounding of answers and checking reasonableness.

Applied problems can be any that are used for addition, subtraction, multiplication or division of whole numbers, by substituting decimal numbers. Addition and subtraction of decimals through hundredths was introduced in 4<sup>th</sup> grade, as was multiplication and division of decimals by whole numbers. 5<sup>th</sup> grade provides more practice with these operations, and extends multiplication to whole numbers x decimals.

The traditional algorithm for multiplying by a decimal number is developed by having students estimate the product first, by recognizing the size of the decimal number. Then they do the multiplication as if there were no decimal point, then they place the decimal point in the answer where it would be in their approximation. For example,  $5 \times 0.25$  can be estimated as 5 times  $\frac{1}{4}$ , or  $\frac{1}{4}$  of 5, which is less than 5 but greater than 1. Doing the multiplication of  $5 \times 25$  gives 125. The actual product cannot be 125 or 12.5, so it must be 1.25. This leads to the shortcut of counting the decimal places and using that to fix the decimal point after multiplying.

Another 5<sup>th</sup> grade core GLCE is:

- Divide numbers by 10s, 100s, 1,000s using mental strategies.

The mental strategy for dividing by 10s, 100s and 1000s is to first divide the numerator by the number of 10s, 100s or 1000s, then create a decimal. For example:

$56 \div 100 = 56/100 = 0.56$  (by the definition of decimal numbers). So  $56 \div 600$  is found dividing 56 by 6 first, which is 9, then dividing 9 by 100,  $9/100 = 0.09$ .

There are two other 5<sup>th</sup> grade GLCEs related to decimals, although they are “extended” MEAP GLCEs and not tested nearly as heavily as the core GLCEs:

- Multiply a whole number by powers of 10: 0.01, 0.1, 1, 10, 100, 1,000 and identify patterns.
- Multiply one- and two-digit whole numbers by decimals up to two decimal places.

Another approach to multiplying by a decimal number involves the GLCE about multiplying a whole number by powers of 10 and identifying patterns: The pattern involves the placement of the decimal point:

		start here	go up next		
0.01	0.1	1	10	100	1000
0.56	5.6	56	560	5600	56,000

## Convert measurements within a given system

The “core” GLCE is “Convert measurements of length, weight, area, volume, and time within a given system using easily manipulable numbers.” There is a lot of multiplication and division practice involved with conversions, especially within non-metric units. But there is also a lot of content to learn in the names of the measuring units. This content has to be learned over time, by actually measuring things repeatedly with appropriate measuring tools and making conversions. Students also need to learn *why* the conversions for area and volume are different from those for length.

unit	traditional U.S. system	metric system
weight	1 pound = 16 ounces	(mass) 1 kilogram = 1000 grams
length	1 foot = 12 inches, 1 yard = 3 feet, 1 mile = 5280 feet	1 meter = 100 cm, 1 cm = 10 mm
area	1 square foot (ft <sup>2</sup> ) = 144 square inches (in <sup>2</sup> ) (recognizing that the conversion factors are different from “regular” inches and feet)	(conversions between square meters and sq. cm is not typical since 1 sq. m = 10,000 sq. cm)
volume	1 cubic yard (yd <sup>3</sup> ) = 27 cubic feet (ft <sup>3</sup> )	1 liter = 1000 milliliters, 1 liter = 1000 cubic centimeters (cm <sup>3</sup> or cc)
time	1 hour = 60 minutes, 1 minute = 60 seconds	(same)

## Angles

Measuring angles with protractors is tricky for most students, so they need a lot of practice. Start with angles that are multiples of 10.

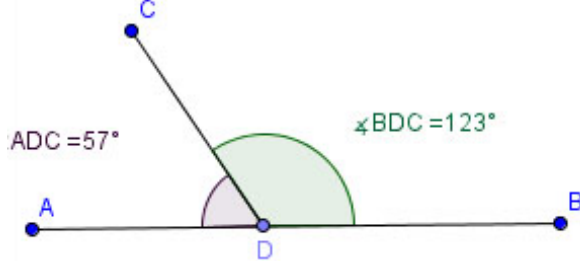
We usually ask students to make up their own classification systems first, whenever we introduce a new taxonomy, but in this case, there are only four classes:

- acute, less than  $90^\circ$
- right, equal to  $90^\circ$
- obtuse, greater than 90 but less than  $180^\circ$
- straight, a straight line, equal to  $180^\circ$

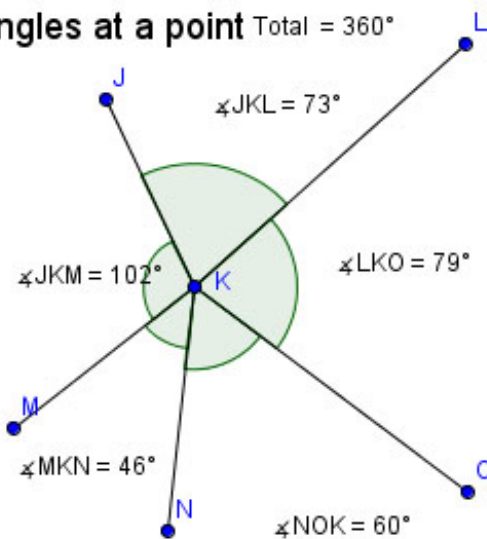
Angles on a straight line (“straight angle”) add up to  $180^\circ$ . Angles surrounding a point add up to  $360^\circ$ . Students are often asked to “find the missing angle” when one angle is given on a straight line, or all but one are given surrounding a point.

### Angles on a straight line

$$\angle ADC + \angle BDC = 57^\circ + 123^\circ = 180^\circ$$



### Angles at a point Total = $360^\circ$



Another “extended” GLCE asks students to identify vertical angles, so these may also be used in unknown angle problems.

### Vertically opposite angles

$$\angle HIF = \angle EIG = 93^\circ$$

$$\angle FIG = \angle EIH = 87^\circ$$

