GLCEs tested on 7th grade MEAP (from the 6th grade GLCEs)

GLCEs in grey will not be tested on 2012-2014 MEAP assessments because they have moved to a higher grade in the Common Core State Standards.

The "core" GLCEs are tested most heavily -2 items per test on every form of the test. The others are "extended" GLCEs and are sampled with 1 item on random forms of the test. The GLCE descriptors below are abbreviations of the actual GLCEs.

Critical areas in 6th grade mathematics include:

- 1) Introduction to algebra, including translating word situations into expressions and equations, simplifying expressions, and solving simple equations
- 2) Using ratios and rates; finding percentages
- 3) Multiplying and dividing fractions (some teaching resources are attached to this document)
- 4) Finding volume and surface area of rectangular prisms

GLCE	GLCE Grade	CALC	GLCE Descriptor	Match to CCSS	
A.PA.06.01	6th	Υ	Solve applied problems involving rates	6th	core
A.RP.06.02	6th	Ν	Plot ordered pairs of integers	6th	
A.FO.06.03	6th	Ν	Use letters, with units, to represent quantities	6th	core
A.FO.06.04	6th	Ν	Distinguish between algebraic expression/equation	6th	core
A.FO.06.05	6th	Ν	Use conventions for writing algebraic expressions	6th	
A.FO.06.06	6th	Υ	Represent words using algebraic equations	6th	core
A.FO.06.07	6th	Ν	Simplify linear expression & evaluate using values	6th	
A.RP.06.08	6th	Υ	Relationships can be shown by graphs and tables	6th	
A.PA.06.09	6th	Υ	Solve problems involving linear functions	6th	
A.RP.06.10	6th	Υ	Show relationships using equations, tables, graphs	6th	
A.FO.06.11	6th	N	Relate simple linear equations to contexts; solve	6th	core
A.FO.06.12	6th	Ν	Add, subtract numbers on both sides of equations	6th	core
A.FO.06.13	6th	Ν	Multiply, divide numbers on both sides of equations	6th	core
A.FO.06.14	6th	Y	Solve equations of the form $ax + b = c$	6th	
D.PR.06.01	6th	Y	Express probabilities as fractions, decimals or %s	7th	
D.PR.06.02	6th	Y	Compute probabilities of events from experiments	7th	
G.GS.06.01	6th	Ν	Understand and apply properties of lines and angles	7th	
G.GS.06.02	6th	Ν	Understand congruence for polygons	8th	
G.TR.06.03	6th	Ν	Understand rigid motions & relate to congruence	8th	
G.TR.06.04	6th	Ν	Use simple compositions of rigid transformations	8th	
M.UN.06.01	6th	Y	Convert measures within a single system	6th	
M.PS.06.02	6th	N	Draw patterns for rectangular prisms	6th	core
M.TE.06.03	6th	Y	Compute volume & surface area of rectangular prisms	6th	core

N.MR.06.01	6th	Ν	Understand \div of fractions as the inverse of x	6th	core
N.FL.06.02	6th	Ν	Write a statement to represent dividing fractions	6th	core
N.MR.06.03	6th	Ν	Solve for the unknown in equations	6th	core
N.FL.06.04	6th	Ν	x and ÷ any two fractions, including mixed numbers	6th	core
N.ME.06.05	6th	Y	Order rational numbers and place on the number line	6th	
N.ME.06.06	6th	Υ	Show rationals as fractions or terminating decimals	6th	
N.ME.06.07	6th	Ν	Understand fractions as a quotient of two integers	6th	
N.MR.06.08	6th	Ν	Understand - and ÷ as inverse of + and x	7th	
N.FL.06.09	6th	Ν	Compute with integers, use # line & chip models*	6th	core
N.FL.06.10	6th	Ν	Compute with positive rational numbers	7th	core
N.ME.06.11	6th	Υ	Find equivalent ratios by scaling up or down	6th	core
N.FL.06.12	6th	Ν	Calculate part of a number given the % and number	6th	core
N.MR.06.13	6th	Υ	Solve contextual problems involving percentages	6th	
N.FL.06.14	6th	Υ	Estimate calculations involving rational numbers	6th	core
N.FL.06.15	6th	Y	Solve applied problems with appropriate decimals	6th	core
N.ME.06.16	6th	Y	Use integer exponents & scientific notation	8th	
N.ME.06.17	6th	Ν	Locate negative rational numbers on number line	6th	
N.ME.06.18	6th	Ν	Understand that rationals are quotients of integers	6th	
N.ME.06.19	6th	Ν	Understand that 0 is neither negative nor positive	6th	
N.ME.06.20	6th	Ν	Know the absolute value of a number	6th	

* MDE made a mistake when they included this GLCE in the list to be tested on the MEAP. Computing with integers has moved to 7th grade in the Common Core, so it shouldn't be tested until 8th grade. MDE is trying to work things out so that those items are not counted in the score.

Multiplying Fractions – Teacher Background Information

An instructional sequence for learning how to multiply with fractions generally takes students through three steps:

1. Multiplying a fraction by a whole number – repeated addition of the fraction

 $4 \cdot \frac{1}{6}$ means 4 groups of $\frac{1}{6}$, or $\frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6}$, which equals 4 sixths, or $\frac{4}{6}$.

2. Multiplying a whole number by a fraction - taking a part of a whole number

 $1/2 \cdot 4$ literally means "1/2 groups of 4" or just "1/2 of 4," which is found by dividing 4 equally into 2 parts, or 4÷2. To find 2/3 of 9, we first find 1/3 of 9 (3) and then we want 2 of them (6).

3. Multiplying a fraction by a fraction – taking a part of a fraction

 $1/2 \cdot 6/7$ means 1/2 of 6/7, or 1/2 of 6 sevenths, which is 3 sevenths - most easily seen as a drawing.



 $1/2 \cdot 6/7$ can also be represented like this, which by counting, shows 6/14 (which is equivalent to 3/7).

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What real-world problems can be solved using each of these ways to multiply fractions? Match each problem below to a type of multiplication above.

1. A bakery has planned to make cakes today. They need the following ingredients for each cake. They want to bake 12 cakes. How much of each ingredient do they need for all 12 cakes?

3/4 cup of sugar	2 1/3 cups of flour
1/4 teaspoon of salt	2/3 tablespoon of baking powder

2. You have 6 donuts and you want to give 2/3 of them to a friend and keep 1/3 for yourself. How many donuts would your friend get? That is, how much is 2/3 of 6?

3. A pan of brownies was left out on the counter and 1/4 of the brownies were eaten. Then you came along and ate 2/3 of the brownies that were left. How much of the whole pan of brownies was eaten?

How can you calculate answers to multiplications involving fractions?

1. When you multiply a fraction by a whole number, you can see that you multiply the whole number times the numerator, and leave the denominator as it is. This is how you calculated $4 \cdot 1/6$ – you're calculating 4 times 1 sixth, which is 4 sixths.

2. When you find a fraction of a whole number, you divide the whole number by the denominator. If the numerator is more than 1, you then multiply the answer by the numerator. This is how you found 2/3 of 9. Of course, you could multiply first, then divide.

3. When you calculate a fraction of a fraction, you can easily see from the example and others like it that you can multiply the numerators and multiply the denominators.

Estimation when multiplying with mixed numbers

Students often find it helpful to estimate answers when finding a fraction of a mixed number, as a first step – just to know what the "ballpark" is for the answer. For the problem $\frac{1}{2} \times 5\frac{3}{4}$,

 $\frac{1}{2}$ of 5 is $2\frac{1}{2}$, a good estimate.

This might lead to the idea that they can find the fraction of the whole number separately from finding the fraction of the fraction, then add the results (the distributive property).

$$\frac{1}{2} \times 5\frac{3}{4} \rightarrow (\frac{1}{2} \times 5) + (\frac{1}{2} \times \frac{3}{4}) \rightarrow (\frac{5}{2}) + (\frac{3}{8}) \rightarrow (\frac{20}{8}) + (\frac{3}{8}) \rightarrow (\frac{23}{8}) = 2\frac{7}{8} \quad \text{(which is close to the estimate of } 2\frac{1}{2})$$

Or they can use similar graphical methods that they developed when finding a fraction of a fraction.

Or a student might decide to convert the mixed number to an improper fraction first. $\frac{1}{2} \times \frac{23}{4}$

Dividing Fractions – Teacher Background Information

An instructional sequence for learning how to divide with fractions is similar to the one for multiplying.

1. Dividing a fraction by a whole number - "Partitioning" into equal groups

 $1/4 \div 2$ means to start with 1/4 of something and divide that quantity equally into 2 groups – in this case, with 1/8 in each group.

2. Dividing a whole number by a fraction - "Measurement division"

 $6 \div 1/2$ means "How many 1/2's are in 6?" Since there are 2 halfs in each whole, times six wholes, there are 12 halfs in 6 wholes. (This is a simple origin of "invert and multiply.")

3. Dividing a fraction by a fraction – Also a case of measurement division

 $3/4 \div 1/4$ means "How many 1/4's are in 3/4?" A simple drawing can show that there are 3 fourths in 3/4.



This is a little trickier when there isn't an integer number of the divisor in the dividend.

 $3/4 \div 1/2$ means "How many 1/2's are in 3/4?" You can see from the drawing above that there is one full 1/2 in the shaded 3/4, plus another half of a 1/2. The answer is 1 1/2. This means there are 1 1/2 halves in 3/4.

If you use the traditional procedure for calculating the answer (invert and multiply) you will get the same answer.

It's important for students to learn how to estimate answers to division problems to develop a sense of what the answer should be.

Multiplying Fractions – Student Workbook

Part 1 Multiplying a fraction and a whole number: Repeated addition

A dime is 1/2 inch wide. If you put 5 dimes end to end, how long would they be from beginning to end?



Write a mathematical expression that shows how you got your answer.

Use fraction circles to figure out 3 times 1/4.

Do the same for these:

$$5 \cdot \frac{1}{8} \qquad \qquad 3 \cdot \frac{2}{5}$$
$$4 \cdot \frac{3}{10}$$

What patterns do you see in your results?

The patterns you see in your results are a prediction. Come up with your own examples to see if your prediction holds.

Find the following. Use manipulatives or drawings to help, if you want.

$$7 \cdot \frac{2}{5} \qquad \qquad 4 \cdot \frac{1}{2}$$

Write the answers to both problems above as improper fractions **and** mixed numbers.

1. A bakery is making cakes today. They want to bake 12 cakes. They need the following ingredients for each cake. How much of each ingredient do they need for all 12 cakes?

For 1 cake	For 12 cakes
$\frac{3}{4}$ cup of sugar	
$\frac{1}{4}$ teaspoon of salt	
$\frac{2}{3}$ tablespoon of baking	
$2\frac{1}{3}$ cups of flour	

Write a story to go with $5 \cdot \frac{1}{8}$

Part 2 Finding a fraction of a whole number

- 2. You have 10 cookies and want to give 1/2 of them to a friend. How many do you give to your friend?
- 3. You have 8 donuts and you want to give 1/4 of them to a friend. How many donuts would your friend get?



(You can think of this as sharing the donuts equally among 4 friends. How many does each friend get?)

We talk about these two problems as 1/2 of 10 and 1/4 of 8. These are multiplication problems, like saying that 6×3 means 6 groups of 3.

You can write the donut problem as $\frac{1}{4}$ of 8 or $\frac{1}{4} \cdot 8$

> How much is
$$\frac{1}{8} \cdot 16$$
, or 1/8 of 16?

Can you see this problem as a division problem? What division expression would represent this problem?

Write an explanation of a mathematical procedure you can use to multiply a unit fraction times a whole number.

If you use repeated addition to figure out 8 · ¹/₄, what do you get?
Is this the same as ¹/₄ · 8 Why?

4. You have 6 donuts and you want to give 2/3 of them to a friend and keep the rest for yourself. How many donuts would your friend get?

Draw a picture if it helps. Explain how you found your answer.

Solution Using what you know, figure out $\frac{3}{4}$ of 12.

When you try each of the multiplications below, say to yourself, this is 3/8 of 16, etc.

$$\frac{3}{8} \cdot 16 \qquad \qquad \frac{2}{5} \cdot 20$$
$$\frac{2}{3} \cdot 9$$

Explain how you found your answers.

- Solve 4/5 of 200 any way you want to, and explain how you did it.
- Make up another problem like this, using a number over 100. Then solve it.
- 5. Five friends buy a package of 12 cookies and want to share them equally. Each friend will get 1/5 of the cookies. How much will each friend get?

Use a drawing to show this if you want.

Part 3 What does it mean to multiply a fraction times a fraction?

6. 3/4 of a pan of brownies was sitting on the counter. You decided to eat 1/3 of the brownies in the pan. How much of the whole pan of brownies did you eat?



You could write this as $\frac{1}{3}$ of $\frac{3}{4}$. Since we know that "of" means "times," this is a multiplication problem: $\frac{1}{3} \cdot \frac{3}{4}$

To figure this out, look at the drawing. What is 1/3 of the shaded area? Color 1/3 of the shaded area with hatch marks. How much of the whole pan of brownies is that?

So what is $\frac{1}{3} \cdot \frac{3}{4}$? This is often confusing, because you have to think about how much of **the whole** you get.



You could visually see the answer to the last two problems in the drawing. The next ones are not as easy, but drawings can help. Here's how you can find $1/2 \cdot 3/4$ (1/2 of 3/4)

First, make a drawing that shows 3/4.

Actually, make it thick.

Then, to find 1/2 of the 3/4, draw a line across the 3/4 to cut it in half.

Shade 1/2 of 3/4 darker.

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Next, ask yourself, how much of the whole is this new, dark shaded part? That is, how much of the whole is 1/2 of 3/4?

Well, when you cut the original 3/4 in half, you made eighths.

1	2	3	4
5	6	7	8

So how many eighths are shaded darkly? This is your answer.

So what is $\frac{1}{2} \cdot \frac{3}{4}$?

Now you try it. Copy this picture onto your paper, then draw lines to find 3/4 of 2/5.

 $\succ \frac{3}{4} \cdot \frac{2}{5}$

Do you see a pattern in these answers? Without making a drawing, predict what the answer would be to this: $\frac{2}{5} \cdot \frac{2}{3}$

Then check your prediction with a drawing.

Write a few sentences to explain to someone else how to multiply fractions:

The method of dividing a rectangle in both directions can be generalized to a procedure. The procedure is to multiply the denominators to find the new denominator, and multiply the numerators to find the new numerator.

3	2	6	2 2 4
4	5	$-\frac{1}{20}$	$\frac{-1}{5}, \frac{-1}{3}, \frac{-1}{15}$

Do you see why that works? Consider 2/5 of 2/3. Create a rectangle that shows 2/3. Then divide that 2/3 into fifths. What fraction have you created in the new drawing? You've created 15ths. This is the product of the denominators.



When you shade the 2/5 part of 2/3, you have shaded four 15ths. 4 is the product of the numerators, a 2x2 grid made from 2 parts of the 2/3 by 2 parts of the 2/5.

7. You have 3/4 of a pizza. You want to divide it equally between two friends. How much do you each get?

- a. Write this problem as a division statement.
- b. Solve it by using the procedure we described above.

c. Show how you could get an answer by using fraction circles or drawing a picture.

Part 4 Simplifying fractions when multiplying

8. You have 2/3 of a pumpkin pie left over from Thanksgiving. You want to give 1/2 of it to your sister. How much of the whole pumpkin pie will this be? (Use fraction circles or drawings to figure this out.)

Here are two drawing methods for this problem:



Now find it using the second method



Are your two answers equivalent?

Which answer is simpler? Why do you think it's simpler?

If you use the numerical procedure to find $\frac{1}{2} \cdot \frac{2}{3}$ by multiplying the numerators and multiplying the denominators, you get an answer that isn't as simple as it could be: $\frac{1}{2} \cdot \frac{2}{3} = \frac{2}{6}$. You then can find an equivalent fraction with smaller numbers, from your knowledge of fraction families: $\frac{2}{6} = \frac{1}{3}$. This is what you get using the first method. (We called this procedure for finding equivalent fractions "scaling down" because the numerator and denominator are proportionally smaller numbers. They have been scaled down by a factor of 2.) Looking at the drawing for the second method, do you see how that is also $\frac{1}{3}$ of the whole? You could move one of the darker colored squares below the other one, then it would show $\frac{1}{3}$.

Try this with a different problem:

- > Use a drawing to find $\frac{3}{4} \cdot \frac{4}{5}$
- > Then find the answer using the numerical procedure.

How are the two answers to this problem similar?

> Try this: $\frac{1}{2} \cdot \frac{4}{9}$ Use a drawing

> Then find the answer using the numerical procedure.

The first method uses a drawing to find the answer. 1/2 of 4/9 means 4/9 divided into two equal groups, or 4/9 divided in half. From the drawing, you can see that you're taking 1/2 of the 4 shaded parts, or dividing the 4 by 2. Then you get 2/9. Looking at the original problem, divide the 4 (in the numerator of the second fraction) by the 2 (in the denominator of the first fraction), to get 2 over 9, or $\frac{2}{9}$.

So an equivalent numerical procedure is to divide one of the numerators by the other denominator to simplify the result. Try this numerical procedure with these multiplications. Use drawings to check your answers.

$\frac{1}{3} \cdot \frac{6}{7}$	$\frac{1}{5} \cdot \frac{20}{27}$
$\frac{1}{4} \cdot \frac{8}{9}$	$\frac{2}{3} \cdot \frac{9}{10}$

Part 5 Working with mixed numbers

Estimate the answer to $\frac{1}{2} \cdot 5\frac{3}{4}$

Now find the actual answer, using any method. Show how you found your answer.

Write a real world situation to go along with this multiplication.

Multiply $2\frac{3}{4} \cdot \frac{2}{3}$ Show how you found your answer.

A general procedure for finding answers to these multiplications is to change the mixed number into an improper fraction, then use the procedure of multiplying numerators and multiplying denominators. Try it with the two problems above (if that's not what you did already). Does it give the same answers?

Dividing Fractions – Student Workbook

Part 1 Dividing a fraction by a whole number

One definition of division is "partitioning" a number into equal groups. With whole numbers, this means " $6 \div 2$ " is 6 partitioned into 2 groups – with 3 in each group.



The partitioning division question is "How many are in each group, if we make 2 equal sized groups?" The procedure is to start passing out the 6 items into 2 groups until all are passed out, and then count the number in each group. We have divided 6 evenly into 2 groups.

It means the same thing with fractions. " $2/4 \div 2$ " means to start with 2/4 of something and divide that equally into 2 groups. For example: You have 2/4 of a pizza and you want to share it equally between 2 people. How much of the pizza does each person get? Make a drawing to show this.

Try these:

$$\frac{4}{12} \div 2 \qquad \qquad \frac{4}{5} \div 2 \qquad \qquad \frac{6}{5} \div 3$$

This problem is just a little harder: You have 1/4 of a pizza and want to share it equally between 2 people. How much does each person get?

Try these:

$$\frac{3}{4} \div 2 \qquad \qquad \frac{3}{8} \div 2 \qquad \qquad \frac{10}{8} \div 3$$

What have you discovered about dividing a fraction by a whole number?

Part 2 Dividing a whole number by a fraction

Write a story to go with this problem: $12 \div 1/2$. We'll come back to it later.

The other definition of division is called "measurement division." It asks the question, how many of the divisor are in the dividend. In the case of $6 \div 2$, ask: How many groups of 2 are in 6? The answer (of course, again) is 3.

Dividing a whole number by a fraction is a case of measurement division. "How many of the fraction are in the whole number?" So $6 \div 1/2$ means "How many 1/2's are in 6?" Use circle fractions if you want to figure this out, or a bar model.

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Interestingly, the answer to $6 \div 1/2$ is the same as the answer to $6 \cdot 2$. (Why?) Try the following:

$0 \cdot 1/2$ $0 \cdot 1/1$ $0 \cdot 2/3$	8 ÷ 1	/2	$6 \div 1/4$	6 ÷ 2/3
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What procedure might you use for the last problem? Try some other problems like that one to test out your procedure.

1. A baker is making cakes for a big party. She uses 1/4 cup of oil for each cake. How many cakes can she make if she has a bottle of oil that has 6 cups in it?

Write this problem as a division expression.

2. The serving size for the granola that Ted likes to eat for breakfast is 3/4 cup. How many servings are there in a box that holds 13 cups?

Write this problem as a division expression.

Look back at the story you wrote to go with $12 \div 1/2$. Knowing what you know now, would you change it in any way?

Part 3 Dividing a fraction by a fraction

3. How many quarters are in 75 cents?

Write this problem as a division expression.

Write this problem using fractions.

This is also a case of measurement division. The question is "How many $\frac{1}{4}$'s are there in $\frac{3}{4}$? How do you find the answer?

Try these:

4	<u> </u>	4 _ 2	$2^{3} \pm 1$
5	5	$\frac{1}{5}$ $\frac{1}{5}$	$\frac{2}{4} \cdot \frac{1}{8}$

Say each of these division problems out loud, like the previous example: "How many 1/5's are in ..."

4. How many 1/2 cup servings are in a package of cheese that contains 5 1/4 cups altogether?

Write this problem as a division expression.

Draw a picture to help you figure this out. Start with $5\frac{1}{4}$.

1		
	1	
	1	

Draw a line where $5\frac{1}{4}$ is. Then count how many 1/2's are in this amount.

Is the answer a whole number? Is there anything left over? If so, what part of 1/2 is left over? So the answer is:

Try some others by drawing the original amount and asking "How many of the divisor is in the original amount?"



$$3\frac{1}{2} \div \frac{1}{2}$$
 $2\frac{1}{3} \div \frac{1}{6}$

Procedures for dividing fractions

When you have a problem like $\frac{5}{2} \div \frac{1}{2}$ (how many $\frac{1}{2}$'s are there in $\frac{5}{2}$?) you can see that the denominators don't really matter. They are simply the "size of the pieces." You get the same answer for $\frac{5}{4} \div \frac{1}{4}$ or $\frac{5}{8} \div \frac{1}{8}$ or $\frac{5}{3} \div \frac{1}{3}$. The answer is 5.



This represents 5/2. How many 1/2's are there?



This represents 5/4. How many 1/4's are there?

This is the number you get when you divide the numerators (and ignore the denominators).

How about $\frac{6}{10} \div \frac{2}{10}$? (How many $\frac{2}{10}$'s are in $\frac{6}{10}$?) Use the drawing below to figure it out:



You really asked: How many 2's are in 6? (What's 6 divided by 2?) Once again, the size of the denominator doesn't matter because it's just the size of each piece. The answer would be the same if you were talking about fourths or thirds or eighths. The answer is 3.

The procedure is this: When you have a common denominator, simply divide the numerators.

You can make this work when you don't start with a common denominator, by scaling up one fraction to get common denominators. Try it with these problem:

8	. 1	6 _	1
12	$\frac{1}{3}$	4	2

18	<u> </u>	$2\frac{1}{2}$ -	1
8	. 4	$2\frac{1}{3}$	6

You could scale up both fractions to a common denominator if you need to. Try it:

$$\frac{7}{3} \div \frac{1}{4}$$

In this case, do you get a whole number as an answer?

You should recognize #4 and #5 as division problems now. Solve them any way you can.

4. Mrs. Murphy's class is making pillow cases. Each pillow case uses 3/4 of a yard of fabric. How many pillow cases can they make out of 12 1/2 yards of fabric? Will any fabric be left over? If so, how much?

- 5. A book shelf is 3 1/2 feet long. Each book on the shelf is 5/8 inches wide. How many books will fit on the shelf?
- 6. How many candy bars can you buy with \$9.50 if each candy bar costs \$0.75. -or-How many candy bars can you buy with 9 1/2 dollars if each candy bar costs 3/4 of a dollar?

"Invert and multiply"

You've probably heard that you can "invert" the second fraction (the divisor) and multiply the two fractions (multiply the numerators and multiply the denominators).

Try it with $\frac{7}{3} \div \frac{1}{4}$ to verify that you get the same answer as

when you scaled up both fractions to a common denominator and then divided the numerators. Did you see the same processes that you used for scaling up and dividing?

(One process was multiplying 7 times 4. You did this in both procedures. Another was multiplying 3 times 1. The third process was dividing 28 by 3. You do all three processes in both procedures, which is why you get the same answer.)

The reason this old saying works is that multiplication is the opposite of division. So multiplying by the inverse is like doing the opposite twice (like turning left and then turning right – you're going in the same direction again; or like holding up a written word in a mirror and then looking at the reflection in a mirror again – you see the original word).

To review, write what you know about dividing fractions, on a separate piece of paper. Use examples to explain what you know.