Peabody College of Vanderbilt University

HOT MATH:

Teaching Math Problem Solving with Explicit Instruction to Transfer and Self-Regulation Strategies

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ABOUT HOT MATH

Hot Math is a whole class program for third grade classrooms. The Hot Math complete manual includes scripts for teaching these lessons, overheads, worksheets, homework and posters. Everything is contained in it so that teachers can pick up the manual and begin the program Each unit focuses on word problem solving strategies. The program is divided into 5 units: Checking Your Work, Buying Bags, Shopping List, Half, Pictograph.

"Checking your work" focuses on the basics of checking over finished work covering these areas: Sense, Lining Up, Math, Labels, Signs.

"Buying Bags" teaches students specific strategies on how to solve word problems that deal with buying things in groups. (e.g. If lollipops come in bags of 10, and Judy needs 23 lollipops, how many bags does she need to buy?)

The "Shopping List" unit teaches students how to solve multi step problems dealing with buying multiple things at multiple prices (e.g. I need to buy 2 apples for $1 each and 4 bananas for $2 each, how much will I spend?)

The "Half" unit teaches students a specific strategy for finding half of a group of objects.

The "Pictograph" unit teaches students how to 'solve' pictographs and use the info to answer questions.

Included herein is an article on CASL article on Hot Math.

To order, please complete the order form and send with a check or purchase order made out to Vanderbilt University. Purchase orders can be fax'd to Flora Murray at 615/343-1570,

Consultation to come to your school district to train teachers on how to implement Hot Math is available. If you are interested, send an email to flora.murray@vanderbilt.edu and she will forward your request to a trainer.

CASL News
Promoting Success in Grades K-3
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Hot Math *Promoting Mathematical Problem Solving Among Children with Disabilities*

By Lynn S. Fuchs and Douglas Fuchs

Mathematical problem solving requires students to apply knowledge, skills, and strategies to novel problems. This can be difficult to achieve, especially for primary-grade children with disabilities. This feature article describes the third-grade mathematics research, which we call "Hot Math," being conducted at Vanderbilt University.

CASL has integrated two promising procedures to promote mathematical problem solving among third-grade children with disabilities: explicit instruction about transfer and self-regulation strategies.

Explicit Instruction About Transfer

To achieve mathematical problem solving, students must learn three things. First, they must master the rules for solving the types of word problems addressed in the curriculum. Second, they must learn to classify problems into groups that require the same solution. Third, they must learn to connect novel problems to the familiar ones they already know how to solve. Our explicit instruction about transfer is designed to promote these outcomes in the following ways.

Format

CASL's instruction is explicit. Each lesson begins with the teacher showing a problem that is already worked. The teacher explains what the problem is asking, how to solve the problem, and why that solution method makes sense and works. Explanations rely heavily on posters for frequent student reference and are laced with opportunities for choral responding to promote engagement.

Through each lesson, examples gradually become partially worked so students have increasing opportunities to supply parts of answers. Eventually, in each lesson, students complete several problems entirely in pairs, with stronger students helping weaker students. Each lesson ends with students completing one problem independently and checking the answer against a scoring key.

Figure 1

Example of Each of Four Problem Types for Which Solution Rules Are Taught

Shopping List Problem Type

Danny needs to buy things for his science project. He needs 2 batteries, 3 wires, and 4 magnets. The batteries cost $3 each, the wires cost $3 each, and the magnets cost $2 each. How much money does Danny need for his science project?

Half Problem Type

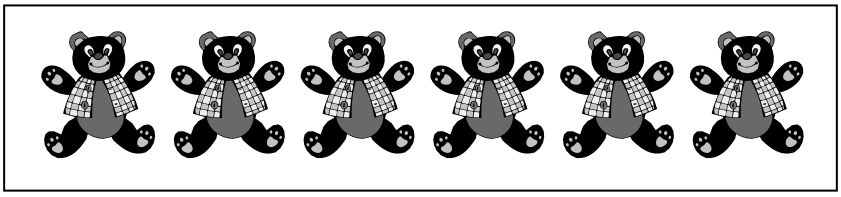
Dave and Todd are going to buy a large box of baseball cards. There are 42 cards in the box. Dave and Todd will each get 1/2 of the cards. How many cards will each of them get?

Bag Problem Type

You want to buy some lemon drops. Lemon drops come in bags with 10 lemon drops in each bag. How many bags of lemon drops should you buy to get 32 lemon drops?

Pictograph Problem Type

Gloria collects teddy bears. She made a chart to show how many teddy bears she had. Each picture of a bear stands for 4 bears.



For her birthday, Gloria got 3 more teddy bears. How many bears does she have now?

Content

The first 3-week unit is dedicated to basic problem-solving information: making sure answers make sense, lining up numbers from text correctly to perform math operations, and labeling work with words and mathematics signs. The next four 3-week units each focuses on one problem type: "shopping list" problems, "half" problems, "buying bags" problems, and "pictograph" problems (see Figure 1 for sample problems). The instructional principles can be applied to other problem types as well.

Each of these four 3-week units provides instruction on skill acquisition and transfer. Sessions 1-2 during the first week and Sessions 3-4 in the second week focus on skill acquisition (what the problem is asking and how to solve the problem), with cumulative review across units incorporated in Session 4. During the third week, two transfer lessons are provided in Sessions 5-6, with cumulative review again incorporated in Session 6. In each unit, the first acquisition lesson (Session 1) and the first transfer lesson (Session 5) last 40 minutes; the others last 25-30 minutes.

To teach rules for problem solution, all problems are structured as shown in Figure 1, but cover stories and quantities vary. A poster listing the steps of the solution method is displayed. In Session 1 of each 3-week unit, teachers present a worked example and, as they refer to the poster, explain each step of the solution method. After presenting several worked

Continued on page 2

CASL reading writing math
National Center on Accelerating Student Learning

examples in this way, teachers move to partially worked examples, where students work in pairs to apply steps of the solution and report answers. Students then complete problems entirely in dyads and check work with answer keys. Finally, students complete one problem independently and then are assigned a homework problem, which they return the next morning to the homework collector (a competent student in the class). *Sessions 2-4* of each unit are structured similarly to Session 1 except that more time is allocated to dyadic practice on problems.

In *Sessions 5-6*, transfer is addressed in three ways. First, teachers explicitly teach the concept of *transfer*. They teach what transfer means: to *move* (i.e., just like we transfer [move] to a different school, we can transfer [move] skills we learn to new situations). They present examples of how children transfer skills; for example, teachers discuss how, as a baby, we learn to drink from a toddler cup, then "move" this skill to a real cup, then "move" this skill to a glass, then "move" this skill to a soda pop bottle. Other examples are presented from everyday life, and children volunteer examples. Teachers also include a math example (i.e., we learn to add 2-digit horizontal problems; then "move" this skill to solve 2-digit vertical problems; then "move" this skill to solve 3-digit problems; then "move" this skill to the check-out counter of the store where we add the cost of two items to figure how much money we need).

After discussing the meaning of *transfer*, teachers teach four types of problem features that can change a problem without altering its structure or solution: A familiar problem structure can be formatted so that the problem looks novel; can use an unfamiliar key word; can pose an additional question; or can be placed within a larger problem-solving context (see Figure 2). A poster listing the four "Ways Problems Can Change" is displayed. In *Session 5*, teachers explain the poster. Then, students classify problems (of that unit's problem type) according to which problem feature has changed and explain how problems seem different but still represent the problem type. Teachers next have students work in pairs to solve transfer problems, which systematically vary one feature at a time. Students complete and check one transfer problem independently, and homework is assigned. *Session 6* is structured similarly to Session 5, except that it also incorporates cumulative review across units.

Figure 2

Superficial Problem Features Taught in the Transfer Treatment
Illustrated with the Bag Problem Type

Original Problem

You want to buy some lemon drops. Lemon drops come in bags with 10 lemon drops in each bag. How many bags should you buy to get 32 lemon drops?

Different Format

- You want to buy some lemon drops.
- The sign at the store looked like this:

LEMON DROPS ON SALE!!!

10 in each bag!!

How many bags should you buy to get 32 lemon drops?

3 4 2 5

Different Key Word

You want to buy some lemon drops. Lemon drops come in packages with 10 lemon drops in each package. How many packages should you buy to get 32 lemon drops?

Additional Question

You want to buy some lemon drops. Lemon drops come in bags with 10 lemon drops in each bag. How many bags should you buy to get 32 lemon drops? If each bag costs $4, how much money will you spend?

Larger Problem-Solving Context

You saved $37. Your friend saved $12. You and your friend want to buy some lemon drops. Lemon drops come in bags with 10 lemon drops in each bag. You want 32 lemon drops.

1. How much money do you and your friend have?
2. If each bag of lemon drops costs $4, how much money will you spend on lemon drops?
3. If you also buy a hat that costs $15, how much money will you have left?

Throughout both transfer sessions of each unit, teachers remind students to search novel problems for "Ways Problems Can Change" to identify familiar problem structures and apply the solutions they know. Teachers also encourage students to look for opportunities during other parts of the school day and outside of school where they might transfer the skill to problems or situations requiring the same solution methods (e.g., other math assignments, homework, grocery shopping).

Helping Students Regulate Their Learning

As reflected in the content of the transfer lessons, mathematical problem solving requires metacognition, whereby children regulate the selection and use of skills and strategies. For this reason, we incorporate the self-regulated learning strategies of *goal setting* and *self-monitoring* throughout the problem-solving transfer treatment. Goal setting helps motivate children to focus their effort and work hard. Monitoring their own progress helps children use their skills and strategies in ways that facilitate goal attainment.

Self-regulated learning strategies are incorporated into each session, with six additional activities.

First, students score the final, independent problem of each session using an answer key that provides credit for the process by which work was completed and the accuracy of the answer.

Second, students graph these daily scores on their personal thermometer chart (Figure 3), which shows consecutive thermometers (one for each session) with each thermometer going from 0 to the maximum score for that problem type.

Third, at the beginning of the next session, students inspect their charts and set a goal to beat their highest score on the day's independent problem.

Fourth, students score their homework prior to submitting it to the homework collector.

Fifth, at the beginning of each session, students report to the class examples of how they have transferred the unit's problem structure to another part of the school day or outside of school.

The sixth activity involves a class graph, on which teachers record the number of students who complete, score, and submit homework and the number of pairs reporting a transfer event to the class.

In these ways, self-regulated learning strategies incorporate goal setting and self-assessment referenced to the content of instructional sessions, including acquisition of problem-solution rules and transfer.

What to Expect

We have explored the effectiveness of CASL's Hot Math mathematical problem-solving program in studies where (a)

Accelerating Student Progress: What We Know

This column provides instructional tips and resources and is a regular feature of *CASL News*.

- When provided with mathematics problem-solving instruction that does not explicitly teach the computational skills required in problems, students with reading and/or mathematics disabilities do not incidentally develop that computational competence. Instead, they require explicit instruction on the computational skills.

Where's the evidence? Fuchs, L.S., Fuchs, D., & Prentice, K. (2002). *Responsiveness to mathematical problem-solving instruction among students with serious mathematics difficulties with and without serious reading difficulties.* Manuscript submitted for publication. Available from Lynn Fuchs.

Want to implement? Contact Lynn Fuchs at *lynn.fuchs@vanderbilt.edu*.

- Building reading fluency can be accomplished with classwide peer tutoring, with the teacher timing students who read aloud simultaneously.

Where's the evidence? Fuchs, D., & Fuchs, L.S. (2002). *Promoting reading fluency with Peer-Assisted Learning Strategies.* Manuscript available from Doug Fuchs.

Want to implement? Contact Doug Fuchs at *doug.fuchs@vanderbilt.edu*.

- The handwriting fluency of students with writing disabilities is likely to be slower than that of students who are good writers. Handwriting fluency can be increased by having children write frequently and via repeated writing–where a child writes the same paragraph several times, trying to gradually increase the number of letters written during a 3-minute time frame.

Where's the evidence? Graham, S., Harris, K., & Fink, B. (2000). Is handwriting causally related to learning to write? Treatment of handwriting problems in beginning writers. *Journal of Educational Psychology, 92,* 620-633.

Want to implement? Contact Steve Graham at *sg23@umail.umd.edu*.

- When teaching reading comprehension, it is important to explain fully to students what to do, and why, how, and when to do it; to model one's own thinking processes; to encourage students to ask questions and discuss possible answers among themselves; and to keep students engaged in their reading by means of providing tasks that demand active involvement.

Where's the evidence? See Williams, J.P. (2002). Reading comprehension strategies and teacher preparation. In A. E. Farstrup & S. J. Samuels (Eds.), *What research has to say about reading instruction* (3rd ed.). Newark, DE: International Reading Association.

Want to implement? Contact Joanna Williams at *jpw15@columbia.edu*.

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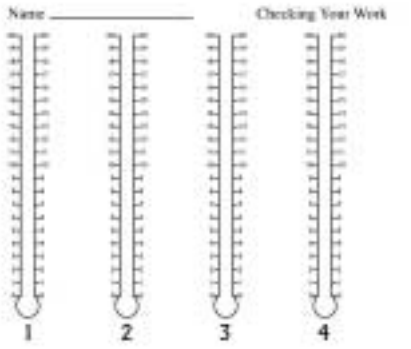
Figure 3 Personal Thermometer Chart



classrooms or children were assigned randomly to control and to different experimental treatments and (b) interventions were implemented for 16 school weeks.

A series of two studies, conducted in mainstream classes with whole-class instruction, showed the specific contribution, for students with and without disabilities, of the innovative components we have incorporated into the program: explicitly teaching for transfer and self-regulated learning strategies. The combined program is very successful for students with and without disabilities. Other studies have shown that the effects of CASL's mathematical problem-solving program are even larger

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From The Teacher's Desk

By Kim Bingham and Karen Mellette
Third-Grade Teachers at Tom Joy Elementary in Nashville, Tennessee

We Love the Vanderbilt Problem-Solving Program at Tom Joy!

The Hot Math Program has really helped our students with their problem-solving math skills. During the lessons, students are excited, motivated, and actively engaged in the learning process. Before implementing Hot Math, problem solving was always something that our students had a hard time mastering, but now because of this well-designed program, our students really learn useful strategies that help them to become successful math problem solvers.

The part of the program that students enjoy the most is keeping track of their progress on their student thermometers. With each skill there are five thermometers. At the end of each lesson, students are given the opportunity to do a problem individually. While working these problems, the students have to be careful and use the strategies presented to them during each skill in order to solve the problems correctly. They are given points for each part they get correct. Their points are then totaled. The fun part for the students is shading in their points on their thermometers. Students are challenged each week to make their thermometers rise higher and higher so they will become Hot in math.

We like how the program teaches different strategies and skills that help the students attack those ever-challenging word problems. One skill that we believe to be useful is called transfer. The students are taught that not all word problems are presented in the same way: some problems can look different, have different words or different questions, or might be a small piece of a bigger problem. Since most of the problems are presented in this way on the state achievement test, being able to transfer enables our students to become better problem solvers in the classroom as well as to perform better on the test.

The students really enjoy this fantastic program! Upon completing the program, they are more confident in their ability to solve a wider variety of math word problems. The Vanderbilt problem-solving program does indeed make students Hot in math.

Continued from page 3

when the program is delivered in small groups rather than in whole-class mainstream settings. When teachers use CASL's program with their students with disabilities, they can expect those students to score more than one standard deviation better than students who receive conventional problem-solving instruction.

How to Obtain CASL's Mathematical Problem-Solving Program

CASL's mathematical program-solving program is available in a manual that includes teaching scripts for implementing all five units and provides all necessary materials (e.g., posters, overheads, worked problems, classroom exercises, scoring keys, homework assignments, personal charts, class charts). To obtain a Hot Math manual, email *mary.b.wilson@vanderbilt.edu*.

For additional information, contact Lynn Fuchs *lynn.fuchs@vanderbilt.edu* or Doug Fuchs *doug.fuchs@vanderbilt.edu* *615-343-4782; fax 615-343-1570*

About CASL News

CASL News provides educators and parents with information to make reading, writing, and mathematics instruction more effective in grades K-3. Issues provide information on the effective instructional practices developed by CASL researchers and their school-based colleagues.

About CASL

The Center on Accelerating Student Learning (CASL) is designed to accelerate learning for students with disabilities in the early grades to provide a solid foundation for strong achievement in the intermediate grades and beyond. CASL is a 5-year collaborative research effort supported by the U.S. Department of Education's Office of Special Education Programs (OSEP). Participating institutions are:

- University of Maryland
- Teachers College of Columbia University
- Vanderbilt University

For a list of CASL research reports, manuals, videotapes, books and other materials, see CASL web site. To order items, send request to:

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